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Specially Structured n-Job 2-machine Flow Shop Scheduling Model With Break-Down Interval And Weightage Of Jobs

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Abstract

This Paper deals with a specially structured n-jobs 2- machine flow shop scheduling problem under specified rental policy in which processing times are associated with their respective probabilities including break-down interval and weightage of job. Further jobs are attached with weights to indicate their relative importance. The objective is to find an algorithm to minimize the rental cost of the machine under Specified rental policy. The method is illustrated with the help of numerical example.

Keywords: Specially structured Flow shop, scheduling processing time, weights of jobs, break-down interval, rental policy.

Introduction

Scheduling has become a major field with in operational research with several hundered. Publications appearing each year. A flow shop scheduling problem has been one of the classical problem in production scheduling since Johnson’s (1954) proposed a well known Johnson’s rule in the two stage flow shop. On specially structured flow shop Smith W.E (1956) considered a special case in which the Job processing times on the first or last machine are the longest and showed that the problem can be solved in polynomial time. The temporal lack of machine availability is known as break-down (due to failure of electric current, non-supply of raw material, shift pattern or other technical interruption.) The work was developed by Ignoble a scourge (1965), Baggu (1969) J.N.D, Yoshida & Hitomi (1979) Singh T.P. (1985), Gupta Deepak (2005) etc by considering various parameters

Gupta Deepak (2012) Shashi bala (2012) studied specially structured n×2 flow shop scheduling with weightage of Job.

This paper is an attempt to extend the study made by Gupta Deepak (2012) by introducing the concept of Break down interval. The concept of break-down interval becomes very significant in the production process where machine while processing the jobs get sudden break-down due to failure of a component of machines for a certain interval of time or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as stop of flow of electric current to the machines may be a government policy due to shortage of electricity production. In each case this may be well observed that working of machines is not continues and is subject interval of time. Hence the problem becomes wider and more applicable in process/ production industries have obtain an algorithm which gives minimum utilization time and hence minimum rental cost.

Practical Situation

Various practical situation occur in real life when one has got assignment but does not have one’s own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent In order to complete the assignment. In his starting career we find a medical. Practitioner does not by expensive machines say x-ray machines, the ultra sound machine. Rotating triple head single position emission computed tomography scanner, patient monitoring equipment and laboratory equipment etc. but instead takes on rent. Rental of medical equipment is an affordable and quick. Solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession Renting enables saving working capital gives option for having the

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equipment, and allows upgradation to new technology. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc.Where various quality of paper, sugar and oil are produced with relative importence i.e weigh in jobs hence weightage of jobs is significance.

Another event which is mostly considered in the models is the break-down of machines. There may also be delays due to material changes in release and tail dates tool unavailability failure of electric current. All of these events complicate the scheduling problem in most cases. Hence the criteria of break-down interval becomes significant. Further the priority of one job over the other may be significant due to some urgency or demand of one particular type of Job over other. Hence the weightage of jobs become important.

Notations

S : Sequence of jobs 1,2,3----------

S_k : Sequence obtained by applying Johnson's procedure K= 1,2,3------

Mj : Machine j, j= 1,2

W_i : Weight of $_i^{th}$ job

A'_{ij} : Weighted flow time $_i^{th}$ job on machine M_j

M : Minimum makespan

aij : Processing time of i^{th} job on machine M_j

P_{ij} : Probability associated to the processing time a_{ij}

A_{ij} : Expected processing time of i^{th} job on machine M_j

L : Length of break-down interval

A''_i : Expected processing time of i^{th} job after break-down effect on machine m_1

B''_i : Expected processing time of i^{th} job after break-down effect on machine M_2

$T_{ij}(s_k)$: Completion time of i^{th} job of sequence S_k on machine Mj

$I_{ij}(s_k)$: Idle time of machine Mj for job i in sequence s_k

$U_j(s_k)$: utilization time for which machine Mj required

$R(s_k)$: Total rental cost for the sequence S_k of all machines

C_i : Rental cost of i^{th} machine

Ct (Si) : Total completion time of jobs for sequence $S_{i.}$

Algorithm

STEP 1: Calculate the expected processing times

$$A_{ij} = a_{ij} \times P_{ij} \quad \forall \ i,j$$

STEP 2 : Compute A'_{i1} and A'_{i2} as follows

1, if min $(A_{ij}) = A_{i1}$ for j= 1,2

Then $A'_{i1} = A_{i1} + W_i$ and $A'_{i2} = A_{i2}$

2, if min $(A_{ij}) = A_{i2}$ for j = 1.2.

Then $A'_{i1} = A_{i1}$ and $A'_{i2} = A_{i2} + W_i$

STEP 3 : Find $A_{ij} = A_{ij}/W_i$

i = 1,2,-----n and j = 1,2

STEP 4: Check the condition

Either $A'_{i1} \geq A'_{i2}$

Or $A'_{i1} \leq A'_{i2}$ for each i

STEP 5: Obtain the job J_1 say having maximum processing time on 1st machine.

STEP 6: Obtain the job J_n (say) having minimum processing time on 2nd machine.

STEP 7: If $J_1 \neq J_n$ then put J, on the first position and J_n as the last position & go to step 11 otherwise go to step 8.

STEP 8: Take the difference of processing times of Job J_1, on M_1 from job J_2 (SAY) having next maximum processing time on M_1 call this difference as G_1 also n take the difference of processing time of job J_n on M_2 from job n J_{n-1} (say) having next minimum processing time on M_2 call the difference.

STEP 9: If $G_1 \leq G_2$ put J_N on last position and J_2 on the first position otherwise put J_1 on 1st position and J_{n-1} on the last position.Arrange the remaining (n-2) jobs between 1st job & lost job in any order thereby we get the sequence S_1, S_2----S_r.

STEP 11: Prepare a flow time table for the sequence obtained in step 7 and read the effect of break- down interval (a,b) on different jobs.

STEP 12: For a redauced problem with processing time A''_i and B''_i as Follow If the break-down interval (a,b) has effect on job$_i$ then

$A'_i = A_{i1} + L$

$B'_i = A_{i2} + L$

Where L = b-a the length of break down interval

If the break-down interval (a,b) has no effect on job i then

$A'_i = A_{i1}$

$B'_i = A_{i2}$

STEP 13: Now repeat the procedure to get the sequence S_i using specially structured two machines algorithm as in step 4.

STEP 14: Evaluate completion time $(t(S_i)$ of 1st job of above selected sequence S_i on machine $M_{1.}$

STEP 15: Calculate utilization time U_i of 2nd machine for each of above selected sequence S_i as

$U_i = t_2(S_i)$-$(T(S_i)$-for i= 1,2,--------

STEP 16: Find min$\{U_i\}$ i= 1,2,------ r let be corresponding to i=m then S_m is the optimal sequence for minimum rental cost.

Min. rental cost = $t_i(S_m) x C_i + U_n \times C_2$

Where C_1 & C_2 are the rental cost per unit time of 1st & 2nd machine respectively.

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Assumptions

1. Jobs are independent to each other let n jobs be processed through two machine M_1 and M_2 in order M_1, M_2.
2. Presumption is not allowed once a job started on a machine the process on that machine cannot be stopped unless job is completed.
3. Either the weighted flow time of $_i^{th}$ job on machine M_1 is longer than the weighted flow time of $_i^{th}$ job on machine M_2 or the weighted flow time of $_i^{th}$ job on machine M_2 for all I i.e either $A'_{i1} \geq A'_{i2}$ or $A'_{i1} \leq A'_{i2}$ for all i
4. Machine break down is also considered.

Theorem

(i) If *for all* $i, j, i \neq j$, then $k_1, k_2 \ldots\ldots k_n$ is a monotonically decreasing sequence, where . $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Proof: Let $A_{i1} \leq A_{j2}$ for all $i, j, i \neq j$
i.e., max $A_{i1} \leq$ min A_{j2} for all $i, j, i \neq j$

Let $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Therefore, we have $k_1 = A_{11}$
Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \leq A_{11}$ $(A_{21} \leq A_{12})$
$\therefore k_1 \leq k_2$
Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$
$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \leq k_2$ $(A_{31} \leq A_{22})$
Therefore, $k_3 \leq k_2 \leq k_1$ or $k_1 \geq k_2 \geq k_3$.
Continuing in this way, we can have $k_1 \geq k_2 \geq k_3 \geq \ldots\ldots \geq k_n$, a monotonically decreasing sequence.

(ii) If $A_{i1} \geq A_{j2}$ for all $i, j, i \neq j$, then $K_1, K_2 \ldots\ldots K_n$ is a monotonically increasing sequence, where . $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Proof: Let $k_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \geq A_{j2}$ for all $i, j, i \neq j$ i.e., min $A_{i1} \geq$ max A_{j2} for all $i, j, i \neq j$
Here $k_1 = A_{11}$
$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \geq k_1$ $(A_{21} \geq A_{j2})$
Therefore, $k_2 \geq k_1$.
Also, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22})$
$= k_2 + (A_{31} - A_{22}) \geq k_2$ $(A_{31} \geq A_{22})$
Hence, $k_3 \geq k_2 \geq k_1$.
Continuing in this way, we can have $k_1 \leq k_2 \leq k_3 \ldots\ldots \leq k_n$, a monotonically increasing sequence.

Problem Formulation

Let some n jobs say i (i = 1,2,---n) are to be processed on two machine M_j (j = 1,2) under the specified rental policy such that no passing is allowed. Let a_{ij} be the processing time of i^{th} job on $_j^{th}$ machine with P_{ij} their respective probabilities such that $\sum P_{i1} = 1 = \sum P_{i2}$; let W_i be the weight of $_i^{th}$ job and break down interval (a,b) is included. Our objective is to find the sequence $\{S_k\}$ of jobs which minimize the utilizaton time and hence rental cost of the machines.

The mathematical model of the problem in matrix form can be stated as:

JOB	Machine M_1		Machine M_2		Weight
I	a_{i1}	P_{i1}	a_{i2}	P_{i2}	W_i
1	a_{11}	P_{11}	a_{12}	P_{12}	W_1
2	a_{21}	P_{21}	a_{22}	P_{22}	W_2
3	a_{31}	P_{31}	A_{32}	P_{32}	W_3
-	-	-	-	-	-
N	a_{n1}	P_{n1}	a_{n2}	P_{n2}	W_n

Table-1

Mathematically the problem is stated as:

Minimize $R(S_k) = U_1(S_k) \times C_1 + U_2(S_k) \times C_2$

Subject to constraint : Rental policy (P) i.e. our objective is to minimize rental cost of machines while minimizing the utilization time.

Rental Policy

The machines will be taken on rent as when they are required and are returned as and when they no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine be taken on rent at time when 1^{st} job is completed on the 1^{st} machine.

Numerical Illustration

Consider 4 jobs, 2 machine flow shop problem with weight of jobs processing tie are associated with their respective probabilities are given in the following table. The rental cost per unit time for machines M_1 and M_2 are 6 units and 8 units respectively and break down interval is (a,b) = (5,10). Our objective is to obtain optimal schedule to minimize the total production time, total elapsed to minimize the total of machine under the rental policy .

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JOB	Machine M_1		Machine M_2		Weight of job
i	a_{i1}	P_{i1}	a_{i2}	P_{i2}	W_i
1	8	0.4	9	0.3	2
2	9	0.2	7	0.2	3
3	11	0.3	30	0.1	4
4	15	0.1	3	0.4	5

Table-2

Solution

As per STEP 1: The expected processing time for machine M_1 and machine M_2 are

JOB	Machine M_1	Machine M_2	Weight of jobs
I	A_{i1}	A_{i2}	W_i
1	3.2	2.7	2
2	1.8	1.4	3
3	3.3	3.0	4
4	1.5	1.2	5

Table-3

As per STEP 2: The new reduced problem with weighted flow time for two machines M_1 and M_2 is

JOB	Machine M_1	Machine M_2
I	A'_{i1}	A'_{i2}
1	1.6	2.35
2	0.6	1.46
3	0.82	1.75
4	0.3	1.04

Table-4

As per step 3:

Here we observed that $A'_{i1} \leq A'_{i2}$ for all i

As per step 4: Max A'_{i1} = 1.6 which is for the 1st job i.e. $J_1 = 1$

Min A'_{i2} = 1.04 which is for the 4th job i.e. $J_n = 4$

Also $J_1 \neq J_n$ on placing J_n on last one the optimal sequences are

S_1 = 1,3,2,4

S_2 = 1,2,3,4

There are two possible optimal sequences. The in-out table for any of these two sequences say S_1 = 1,3,2,4

JOB	Machine M_1	Machine M_2
I	In-out	In-out
1	0 -3.2	3.2-5.9
3	3.2-6.5	6.5-9.5
2	6.5-8.3	9.5-10.9
4	8.3-9.8	10.9-12.1

Table-5

As per step13: On considering the effect of break down interval (5,10) the revised processing times A'_{i1} and A'_{i2} of machine M_1 and M_2 are as follows

L = b-a

= 10-5

=5

JOB	Machine M_1	Machine M_2	Weight of job
I	A''_{i1}	A''_{i2}	w_i
1	6.2	5.7	2
2	1.8	1.4	3
3	6.3	3.0	4
4	1.5	1.2	5

Table-6

As per STEP 2: The new reduced problem with weighted flow time for two machines M_1 and M_2 is

JOB	Machine M_1	Machine M_2
I	A''_{i1}	A''_{i2}
1	3.1	3.85
3	0.45	1.46
2	1.57	1.75
4	0.3	1.24

Table-7

Here we observed that $A''_{i1} \leq A''_{i2}$ for all i
Max A''_{i1} = 3.1 which is for the 1st job i.e. $J_1 = 1$
Min A''_{i2} = 1.24 which is for the 4th job i.e. $J_n = 4$
Also $J_1 \neq J_n$ on placing J_n on last one the optimal sequences are
S_1 = 1,3,2,4
S_2 = 1,2,3,4
There are two possible optimal sequences. The in-out table for any of these two sequences say
S_1 = 1,3,2,4

JOB	Machine M_1	Machine M_2
I	In-out	In-out
1	0 -6.2	6.2-12.9
3	6.2-12.5	12.9-15.9
2	12.5-14.3	15.9-17.3
4	14.3-15.8	17.3-18.5

Table-8

Total elapsed time= $CT(S_1)$=18.5 units
Utilization time for $M_2 = U_2(S_1)$= 18.5-6.2
=12.3 units
Also $\sum A_{i1}$= 15.8
Total rental cost
15.8×10+5.3×7
= 158+37.1=195 units

Remarks

If we solve the above problem by Johnson's (1954) method we get the optimal sequence as
S = 1,2,3,4
The in-out flow table is

JOB	Machine M_1	Machine M_2
I	In-out	In-out
1	0-6.2	6.2-8.9
2	6.2-11.0	11.0-15.4
3	11.0-14.3	15.4-18.4

4	14.3-15.8	18.4-22.6

Table-9

Total elapsed time = CT(S) = 22.6 units

Utilisation time for $M_2 = U_2(S)$

= 22.6-6.2

= 16.4 units

Also $\sum_{i=1}^{n} A_{i=1}$ = 15.8 unitis

Therefor rental cost is R(S) = 15.8×10+16.4×7

= 158+114.8

= 272.8 units

Conclusion

The algorithm proposed in this paper for specially structured two stage flow shop scheduling problem including break-down interval and weightage of job is more efficient as compared to the algorithm proposed by Johnson's (1954) to find an optimal sequence to minimize the utilization time of the machine and hence their rental cost. The study may further be extended by considering various parameters like set up time, transportation time etc.

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